Reverse-Flow Theorem Applied to Subsonic Unsteady Aerodynamic Forces

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Introduction

NUMERICAL methods for predicting generalized unsteady aerodynamic forces usually do not provide identical results. Experimental methods are rarely available to verify the accuracy of predicted generalized forces for complex configuration, and the flutter analyst must rely on calculated results. Convergence criteria generally are not available to enable the user to obtain the best results from a particular numerical method. This important issue was addressed recently by Rowe and Cunningham,[1] and in a Comment by Stark[2] followd by a Reply by Lottati.[3] The usual way to tackle the accuracy of the method is by applying the code on several test cases and comparing these results against results obtained by other methods of computation (see Ref. 1). Another way is to use the existing relations for the aerodynamic forces developed on the configuration in direct and reverse flow. By comparison of the obtained results computed by direct and reverse flow, it is possible to assess the accuracy of the method. The general reverse-flow theorems for lifting-surface theory[4] may also be applied to wings that undergo harmonic oscillation. The results of Ref. 1 emphasize the wavy character of the oscillatory pressure developed on an oscillating wing. The pressure distribution displayed in Ref. 1 aimed to alert users to the possible potential difficulties in obtaining an accurate solution due to the wavy character of the aerodynamic pressure on an oscillating wing. The pressure distribution becomes extremely wavy and more difficult to predict accurately as the reduced frequency k and the Mach number M get larger values. Reference 1 defined a wave number to assess the waviness of the pressure distribution which might be useful in the estimation of relative difficulties in predicting converged aerodynamic forces. In this Note, the reverse-flow theorem will be applied to assess the accuracy of the computed unsteady aerodynamic forces for interference configuration.

The computer program used to evaluate the unsteady aerodynamic forces on the configuration is the nonplanar piecewise continuous kernel function method (PCKFM).[5]

Application of the Reverse-Flow Theorem

Applications of the reverse-flow theorem for nonstationary flow are similar to those for stationary flow. The reverse-flow theorem is formulated as follows:

$$\iint_S w(x,y)\Delta\bar{p}(x,y)\,\mathrm{d}x\mathrm{d}y = \iint_S \bar{w}(x,y)\Delta p(x,y)\,\mathrm{d}x\mathrm{d}y \qquad (1)$$

where $w(x,y)$, $\Delta p(x,y)$, are the downwash and pressure distribution, respectively, on the configuration in direct flow and $\bar{w}(x,y)$, $\Delta\bar{p}(x,y)$ are held the same in the reverse-flow case.

Thus, for a configuration that oscillates in vertical translational mode (heave mode), the oscillatory motion in the direct flow is described by

$$z = He^{i\omega t}, \qquad w(x,y) = V\partial z/\partial x + i\omega z = i\omega He^{i\omega t} \qquad (2a)$$

and for the reversed-flow case by

$$\bar{z} = He^{i\omega t}, \qquad \bar{w}(x,y) = -V\partial\bar{z}/\partial x + i\omega\bar{z} = i\omega He^{i\omega t} \qquad (2b)$$

where H is some representative dimension. Thus, for the heave mode, the lifts in direct and reverse flow are equal (lifts upward are defined as positive). For lifting surfaces that execute pitching oscillation the motion is described by

$$z = xe^{i\omega t}, \qquad w(x,y) = (V + i\omega x)e^{i\omega t}$$

$$\bar{z} = -xe^{i\omega t}, \qquad \bar{w}(x,y) = (V - i\omega x)e^{i\omega t} \qquad (3)$$

Applying the reverse-flow theorem (1), one obtains the following relation

$$L + ikM = \bar{L} + ik\bar{M} \qquad (4)$$

where L and M are the total lift and moment (positive for nose up) of the configuration in direct flow and $\bar{L}$ and $\bar{M}$ are held the same in the reverse-flow case ($k = \omega H/V$ is the reduced frequency). Relation (4) reveals that the lift in the direct and reverse flow of a wing in a constant angle of attack should be equal for the steady flow case ($k = 0$). Thus, the reverse-flow theorem might give an indication of the accuracy of the lift and moment computed for the steady and oscillatory flow for interference configuration.

Results

As stated earlier, the PCKFM computer code was chosen to demonstrate the application of the reverse-flow theorem on interference configurations.

It should be emphasized that the aerodynamic pressure distribution on lifting surfaces in the direct and reverse flow is totally different even though the integral quantities are identical.

To address the accuracy of the computed aerodynamic forces on an oscillating lifting surface, the so-called AGARD configuration is chosen as a test case (see Fig. 1). The horizontal tail of the configuration is placed at different heights above

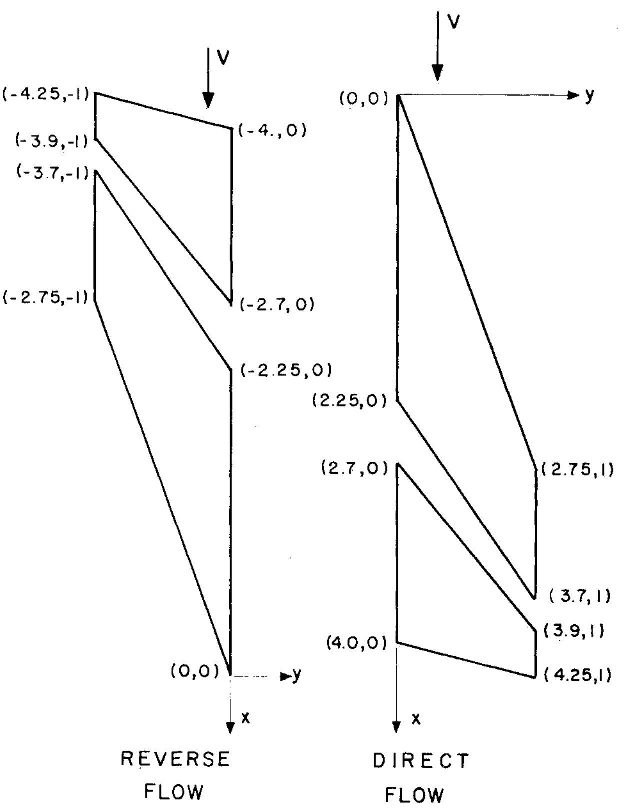

Fig. 1 Plan view of AGARD configuration.

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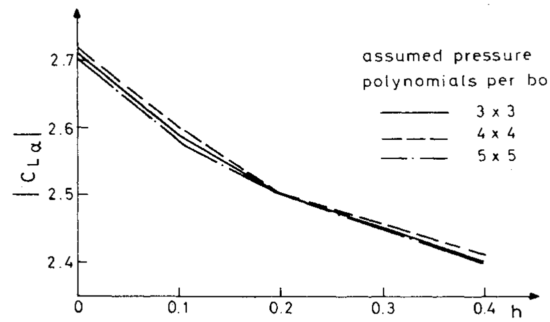
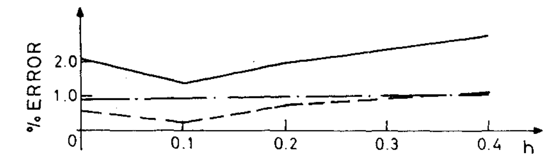
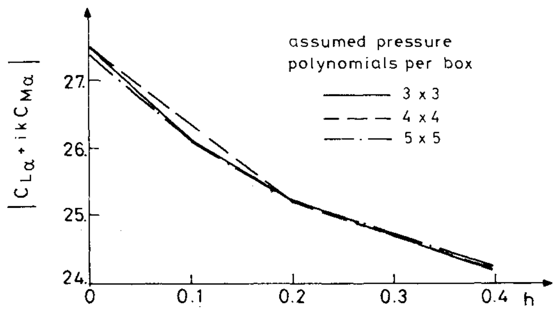


a) Absolute value of lift computed by various combinations of pressure polynomials.

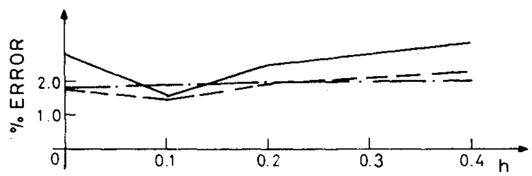


b) Percentage error of the total lift as computed in the direct and reverse flow.

Fig. 2 Convergence study of the total lift for the AGARD interference configuration oscillating in heave ($k=1.0$, $M=0.8$).



a) Absolute value of lift and moment computed by various combinations of pressure polynomials.



b) Percentage error of the total lift and moment as computed in the direct and reverse flow.

Fig. 3 Convergence study of the total lift and moment for the AGARD interference configuration oscillating in pitch ($k=1.0$, $M=0.8$).

the plane of the wing. All the results in this Note were obtained using the PCKFM while representing the AGARD configuration by two boxes (one box each for the wing and the tail).

The aerodynamic lift coefficient (absolute value) is computed for the AGARD configuration undergoing a heave mode oscillating at a reduced frequency $k=1$ (the reference length is equal to unity) and $M=0.8$. Figure 2a represents a convergence study computed assuming different combinations of chordwise and spanwise unknown pressure polynomial coefficients (3×3, 4×4, and 5×5) for different positions of the tail (the height between the wing-tail surfaces is denoted by h). Figure 2b represents the percentage error between the lift (absolute) computed for direct and reverse flow for various combinations of assumed pressure polynomials. It is seen that the error for the lift coefficient computed for the direct and reverse flow is extremely low (1-2%). Figures 3a and 3b show the results of the combination of lift and moment defined by Eq. (4) for direct and reverse flow represented in the same format as that of Fig. 2 (the configuration oscillates in this case in a pitch mode). Assuming that the unsteady lift is computed accurately, as might be concluded from the results of Fig. 2, the results of Fig. 3 indicate that the unsteady aerodynamic moment is also computed fairly accurately (1-2%).

A brief consideration should be given to the relation between the exact air forces (unknown) and the computed aerodynamic forces in the direct and reverse flow (they fulfill the relation of the reverse-flow theorem within acceptable numerical "error"). $w(x,y)$ and $\bar{w}(x,y)$ are prescribed known downwash functions. The PCKFM represents the pressure distribution by orthogonal polynomials[6] to the assumed weight function (in each box). This representation ensures that the error in numerically computing the integrals in Eq. (1) are minimum. By the same token, the aerodynamic forces are computed very accurately. Thus, the exact aerodynamic forces may be assumed to be the average results of the computed air forces in the direct and reverse flow.

Conclusions

Comparisons of predicted aerodynamic lift and moment developed on a configuration in direct and reverse flow indicate that the results obtained by applying the PCKFM code satisfy the condition of the reverse-flow theorem. The reverse-flow theorem can be applied to validate and justify the accuracy of the numerical code used. It is shown that the PCKFM yield converged results for relatively small numbers of assumed pressure polynomials even for fairly high wave numbers. It is suggested that the accuracy of the numerical method should be tested considering the reverse-flow theorem, especially for oscillatory flow. It should be noted that the reverse-flow theorem might be used as a tool to estimate the accuracy of the numerical method even though it cannot be considered as mathematical proof of it.

References

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